Generative Adversarial Network: a Brief Introduction

Lili Mou
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Outline

- Generative adversarial net
- Conditional generative adversarial net
- Deep generative image models using Laplacian pyramid of adversarial networks

NIPS'14

Generative Adversarial Nets

Ian J. Goodfellow[*], Jean Pouget-Abadie[†], Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair[‡], Aaron Courville, Yoshua Bengio[§]
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- Deep generative models are less impactful than deep discriminative models, because...
  - Of the difficulty of approximating many intractable probabilistic computations that arise in maximum likelihood estimation and related strategies
  - Of the difficulty of leveraging the benefits of piecewise linear units in the generative context.
  - (My point of view) Generative problems are much more difficult than discriminative ones.

A Game Theory Perspective

- Two agents:
  - **G**enerative model: Generate new samples that are as similar as the data
  - **D**iscriminative model: Distinguish samples in disguise

- Each agent takes a step in turn

Objective of GAN

$$\min_G \max_D V(D,G) = \underbrace{\mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}(\boldsymbol{x})}[\log D(\boldsymbol{x})] + \mathbb{E}_{\boldsymbol{z}\sim p_{\boldsymbol{z}}(\boldsymbol{z})}[\log(1 - D(G(\boldsymbol{z})))]}_{V(D,G)}$$

- G(z): A generated sample from distribution z
- D(x) = Estimated (by **D**) prob. that x is a real data sample
  - D(x)=1: **D** regards x as a training sample w.p.1
  - D(x)=0: **D** regards x as a generative sample w.p.1
- The relationship with traditional minimax problem
  - In a two-agent WuZi chess, V evaluates how bad the current position is to me.

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Mathematicians seem to be pessimistic creatures who think in terms of losses. Decision theorists in economics and business talk instead in terms of gains (utility).

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  - In a two-agent WuZi chess, V evaluates how bad the current position is to me.
  - Adversary's goal: $\text{maximize} \;\; V$
  - My Goal: $\text{minimize} \;\; \max \;\; V$

Objective of GAN

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Algorithm

for number of training iterations **do**

for k steps **do**

- Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \dots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
- Sample minibatch of m examples $\{\boldsymbol{x}^{(1)}, \dots, \boldsymbol{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\boldsymbol{x})$.
- Update the discriminator by ascending its stochastic gradient:

$\max_D V$

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right)\right].$$

end for

- Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \dots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
- Update the generator by descending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$

end for

Remarks

- Choose k = 1. Recall CD-k
- minimize log(1-D(G(z))) <==> maximize log D(G(z))
- The latter yields a larger gradient especially at beginning steps

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end for

Intuitive Explanation

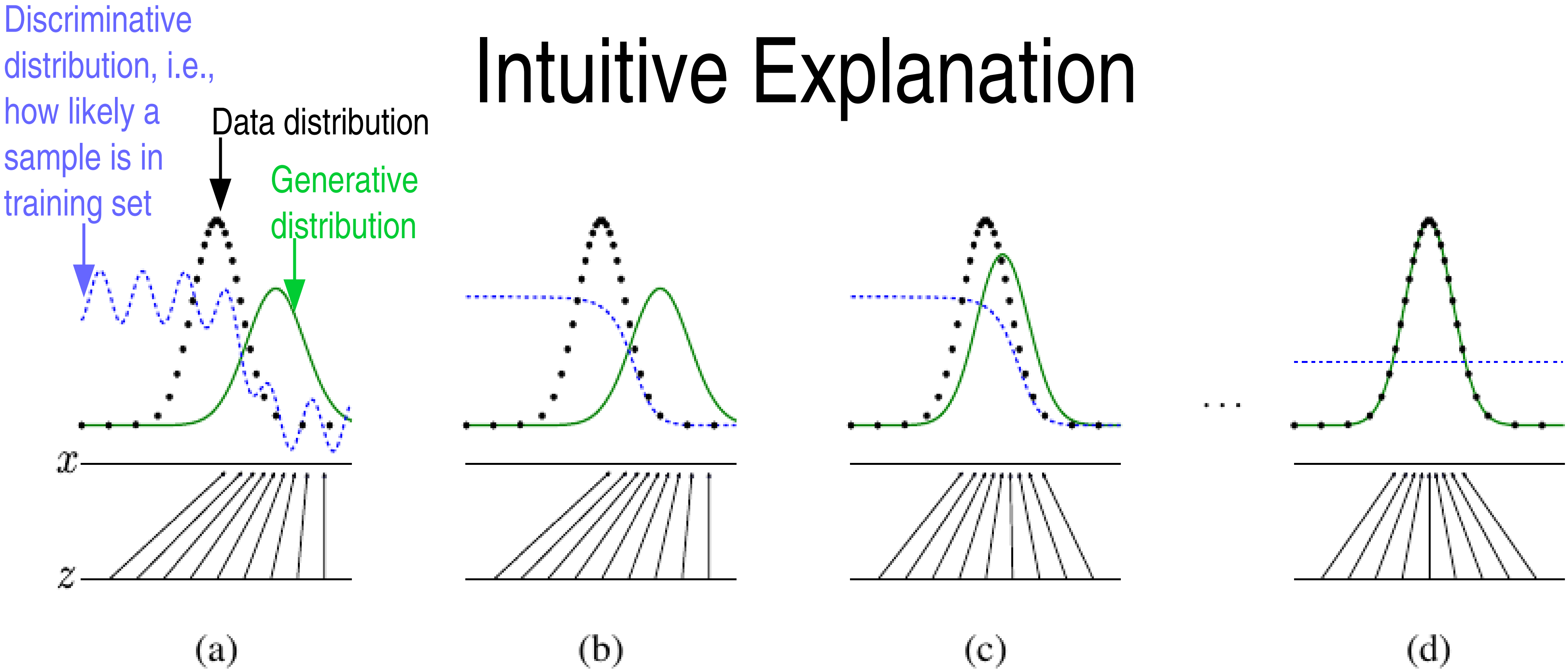

Figure 1: Generative adversarial nets are trained by simultaneously updating the **d**iscriminative distribution (D, blue, dashed line) so that it discriminates between samples from the data generating distribution (black, dotted line) $p_{\boldsymbol{x}}$ from those of the **g**enerative distribution p_g (G) (green, solid line). The lower horizontal line is the domain from which $\boldsymbol{z}$ is sampled, in this case uniformly. The horizontal line above is part of the domain of $\boldsymbol{x}$. The upward arrows show how the mapping $\boldsymbol{x} = G(\boldsymbol{z})$ imposes the non-uniform distribution p_g on transformed samples. G contracts in regions of high density and expands in regions of low density of p_g. (a) Consider an adversarial pair near convergence: p_g is similar to p_{data} and D is a partially accurate classifier. (b) In the inner loop of the algorithm D is trained to discriminate samples from data, converging to $D^*(\boldsymbol{x}) = \frac{p_{\text{data}}(\boldsymbol{x})}{p_{\text{data}}(\boldsymbol{x})+p_g(\boldsymbol{x})}$. (c) After an update to G, gradient of D has guided $G(\boldsymbol{z})$ to flow to regions that are more likely to be classified as data. (d) After several steps of training, if G and D have enough capacity, they will reach a point at which both cannot improve because $p_g = p_{\text{data}}$. The discriminator is unable to differentiate between the two distributions, i.e. $D(\boldsymbol{x}) = \frac{1}{2}$.

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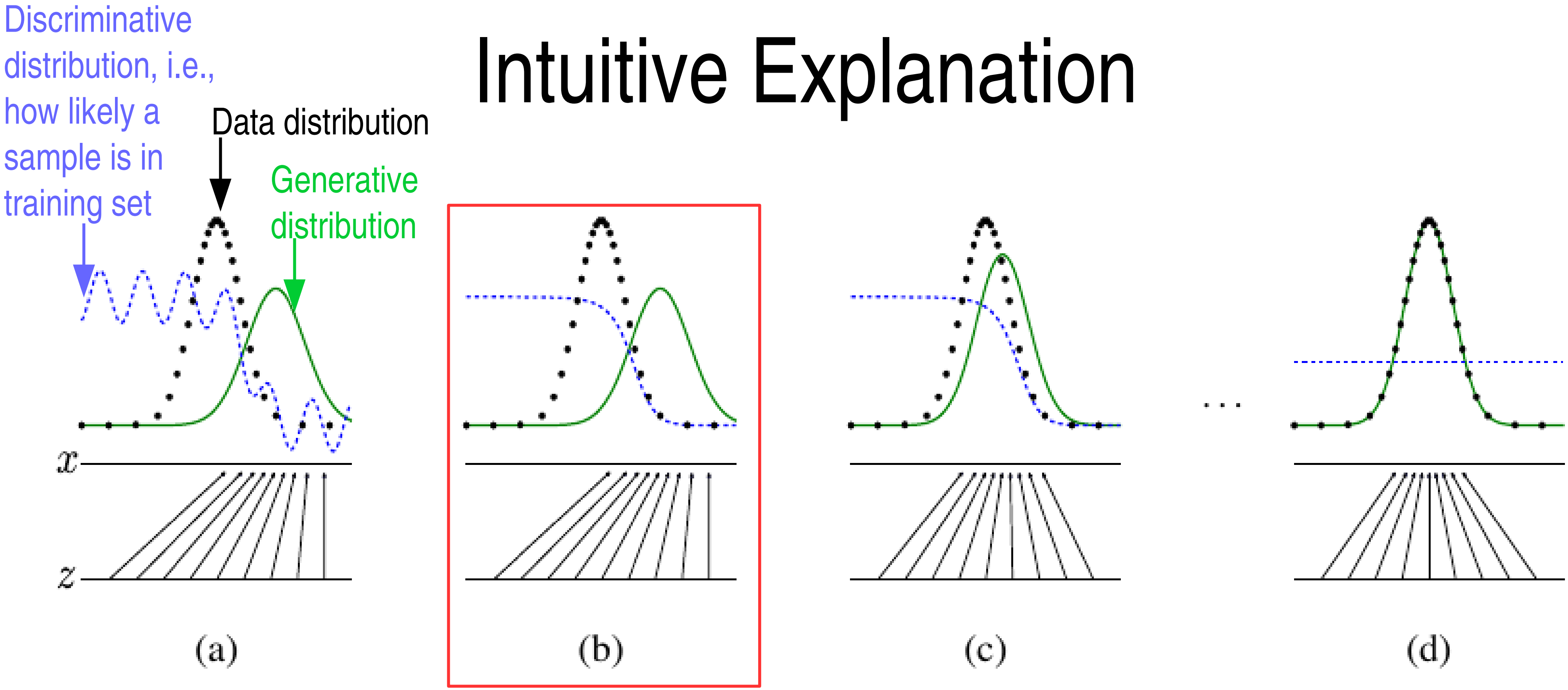

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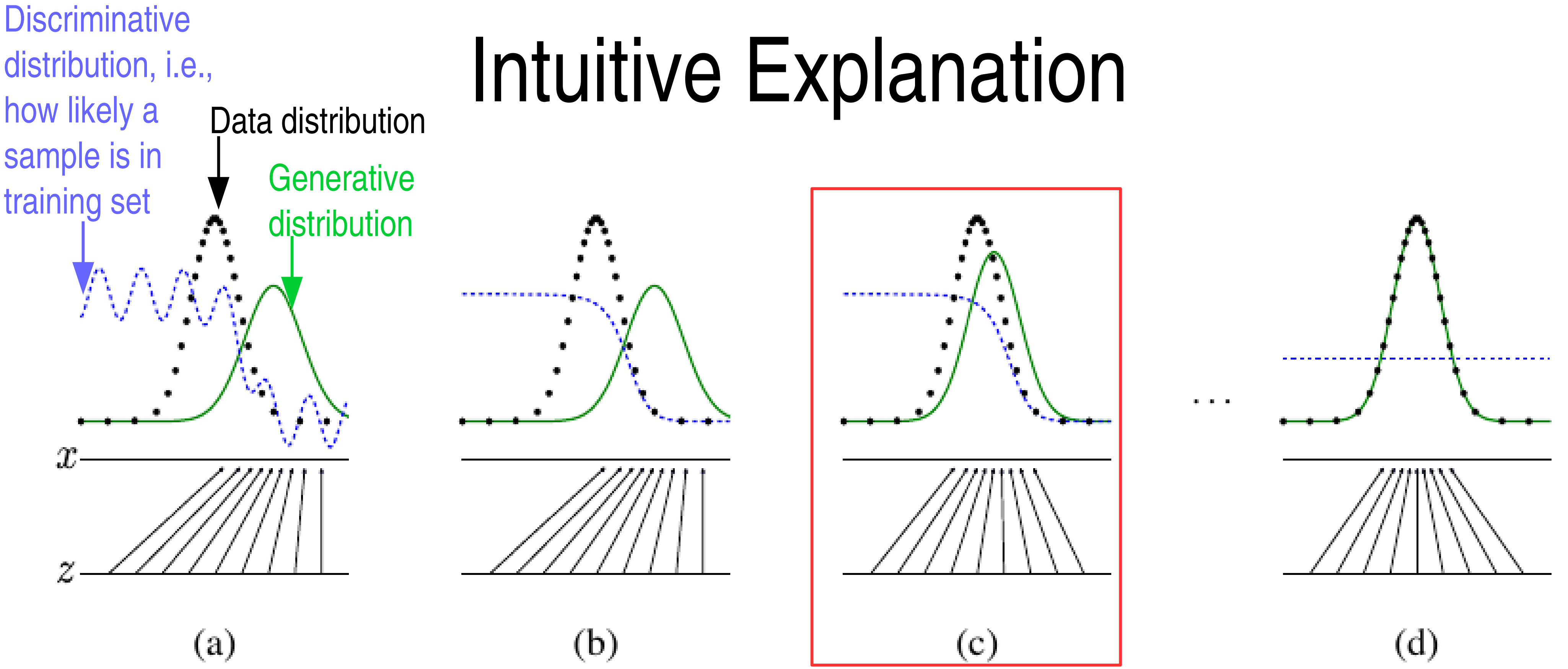
Two-point distribution

Intuitive Explanation



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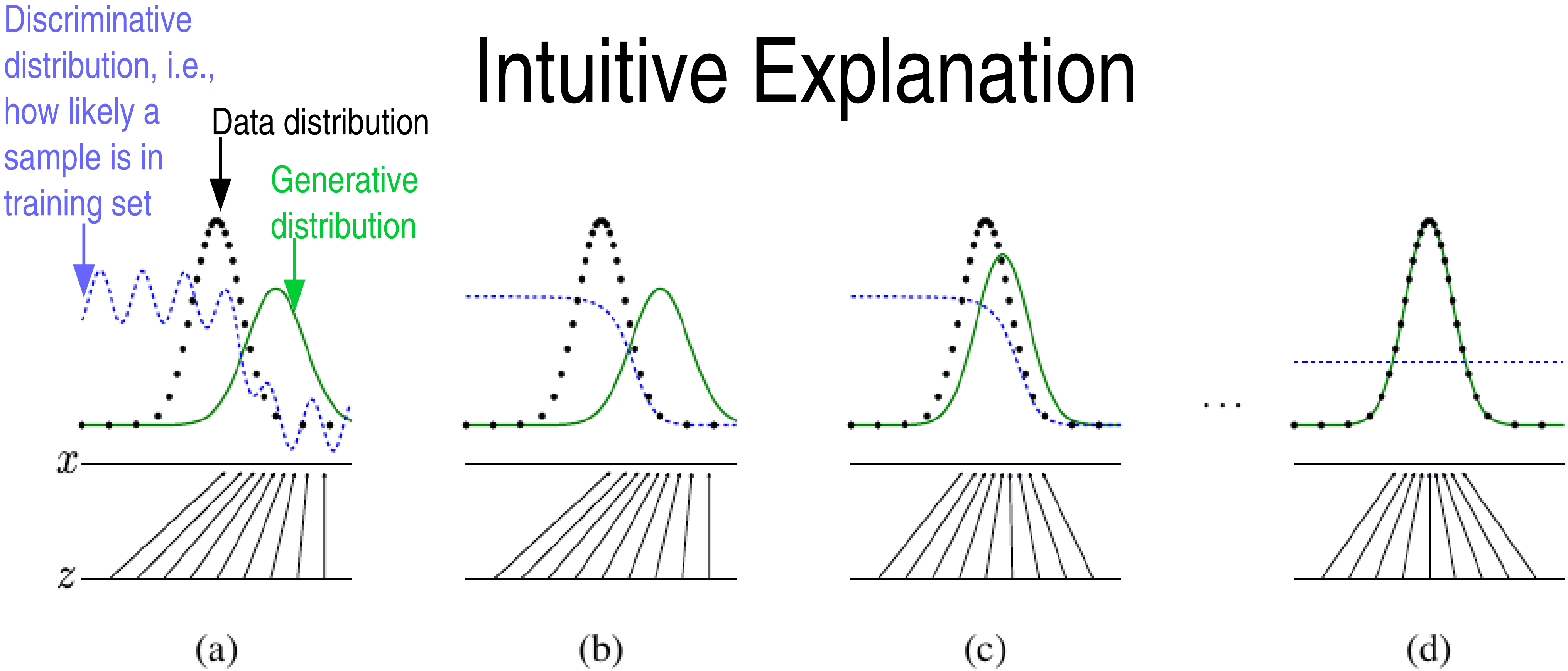

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Theoretical Analysis

4.1 Global Optimality of $p_g = p_{\text{data}}$

We first consider the optimal discriminator D for any given generator G.

Proposition 1. *For G fixed, the optimal discriminator D is*

$$D_G^*(\boldsymbol{x}) = \frac{p_{data}(\boldsymbol{x})}{p_{data}(\boldsymbol{x}) + p_g(\boldsymbol{x})}$$

Proof. The training criterion for the discriminator D, given any generator G, is to maximize the quantity $V(G, D)$

$$\begin{aligned} V(G, D) &= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x}))dx + \int_z p_{\boldsymbol{z}}(\boldsymbol{z}) \log(1 - D(g(\boldsymbol{z})))dz \\ &= \int_{\boldsymbol{x}} p_{\text{data}}(\boldsymbol{x}) \log(D(\boldsymbol{x})) + p_g(\boldsymbol{x}) \log(1 - D(\boldsymbol{x}))dx \end{aligned} \tag{3}$$

For any $(a, b) \in \mathbb{R}^2 \setminus \{0, 0\}$, the function $y \to a\log(y) + b\log(1-y)$ achieves its maximum in $[0, 1]$ at $\frac{a}{a+b}$. The discriminator does not need to be defined outside of $Supp(p_{\text{data}}) \cup Supp(p_g)$, concluding the proof. $\square$

Gobal minimum: p_g = p_data

Theorem 1. *The global minimum of the virtual training criterion $C(G)$ is achieved if and only if $p_g = p_{data}$. At that point, $C(G)$ achieves the value $-\log 4$.*

Proof. For $p_g = p_{\text{data}}$, $D^*_G(\boldsymbol{x}) = \frac{1}{2}$, (consider Eq. 2). Hence, by inspecting Eq. 4 at $D^*_G(\boldsymbol{x}) = \frac{1}{2}$, we find $C(G) = \log \frac{1}{2} + \log \frac{1}{2} = -\log 4$. To see that this is the best possible value of $C(G)$, reached only for $p_g = p_{\text{data}}$, observe that

$$\mathbb{E}_{\boldsymbol{x}\sim p_{\text{data}}}\left[-\log 2\right] + \mathbb{E}_{\boldsymbol{x}\sim p_g}\left[-\log 2\right] = -\log 4$$

and that by subtracting this expression from $C(G) = V(D^*_G, G)$, we obtain:

$$C(G) = -\log(4) + KL\left(p_{\text{data}} \left\| \frac{p_{\text{data}} + p_g}{2}\right.\right) + KL\left(p_g \left\| \frac{p_{\text{data}} + p_g}{2}\right.\right) \quad (5)$$

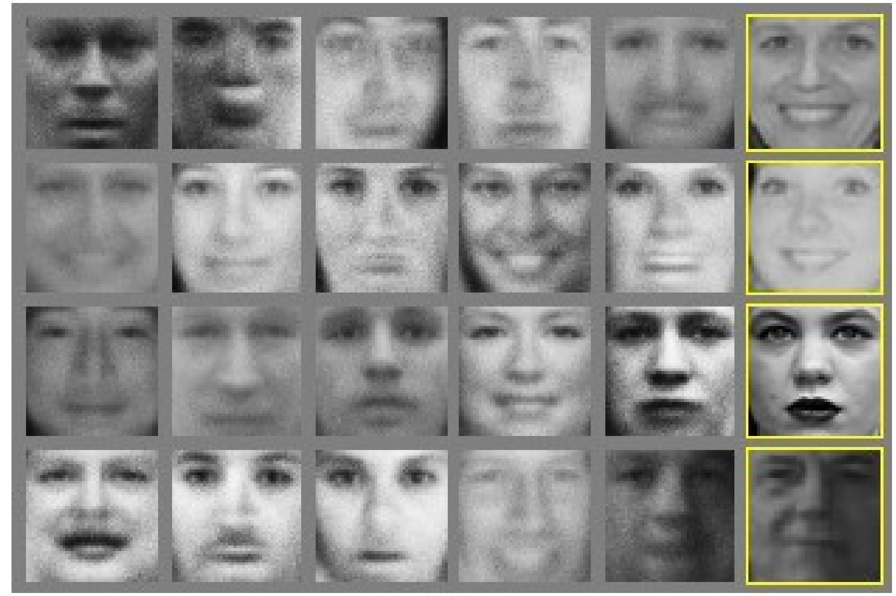
where KL is the Kullback–Leibler divergence. We recognize in the previous expression the Jensen–Shannon divergence between the model's distribution and the data generating process:

$$C(G) = -\log(4) + 2 \cdot JSD\left(p_{\text{data}} \| p_g\right) \quad (6)$$

Since the Jensen–Shannon divergence between two distributions is always non-negative, and zero iff they are equal, we have shown that $C^* = -\log(4)$ is the global minimum of $C(G)$ and that the only solution is $p_g = p_{\text{data}}$, i.e., the generative model perfectly replicating the data distribution. □

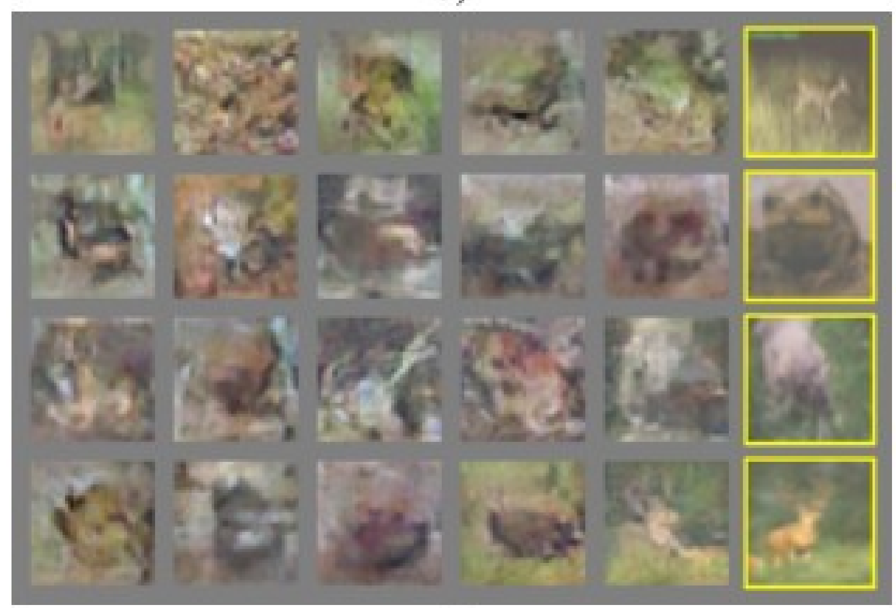
- The cost is convex in p_g.
- But in practice, we always train a parametric model G(z;θg)

a)



b)

c)

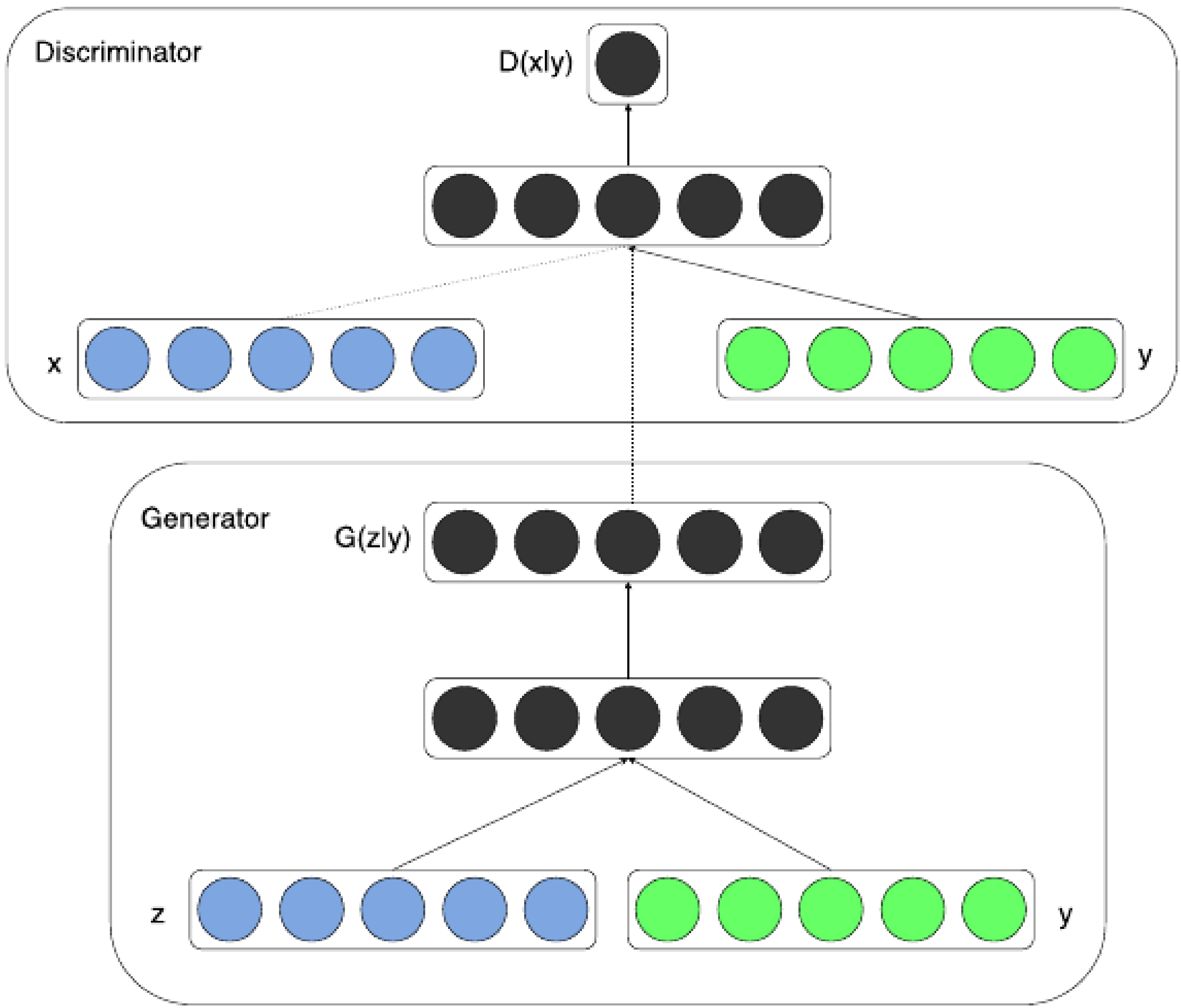


d)

Outline

- Generative adversarial net
- Conditional generative adversarial net
- Deep generative image models using Laplacian pyramid of adversarial networks

Conditional Generative Adversarial Nets



Multi-Modal Generation

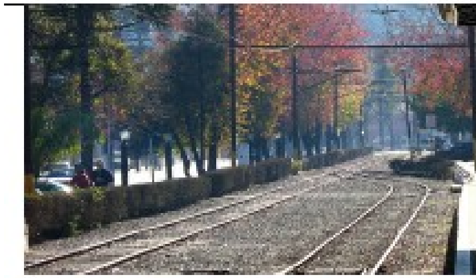
	User tags + annotations	Generated tags
	montanha, trem, inverno, frio, people, male, plant life, tree, structures, transport, car	taxi, passenger, line, transportation, railway station, passengers, railways, signals, rail, rails
	food, raspberry, delicious, homemade	chicken, fattening, cooked, peanut, cream, cookie, house made, bread, biscuit, bakes
	water, river	creek, lake, along, near, river, rocky, treeline, valley, woods, waters
	people, portrait, female, baby, indoor	love, people, posing, girl, young, strangers, pretty, women, happy, life

Table 2: Samples of generated tags

Image convnet, word embeddings pretrained

- **G**:
- **D**:

Multi-Modal Generation

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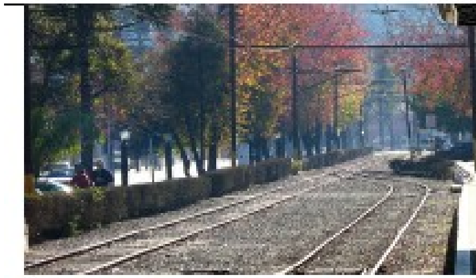	montanha, trem, inverno, frio, people, male, plant life, tree, structures, transport, car	taxi, passenger, line, transportation, railway station, passengers, railways, signals, rail, rails
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Image convnet, word embeddings pretrained

- **G**: Gaussian noise + image features → regression over word embedding
- **D**: image + embedding → sigmoid

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NIPS'15

Deep Generative Image Models using a Laplacian Pyramid of Adversarial Networks

Emily Denton* **Soumith Chintala*** **Arthur Szlam** **Rob Fergus**

Dept. of Computer Science
Courant Institute
New York University

Facebook AI Research
New York

Laplacian Pyramid

- Upsample: an i X i image $\mapsto$ an 2i X 2i image
- Downsample: an i X i image $\mapsto$ an i/2 X i/2 image

Training

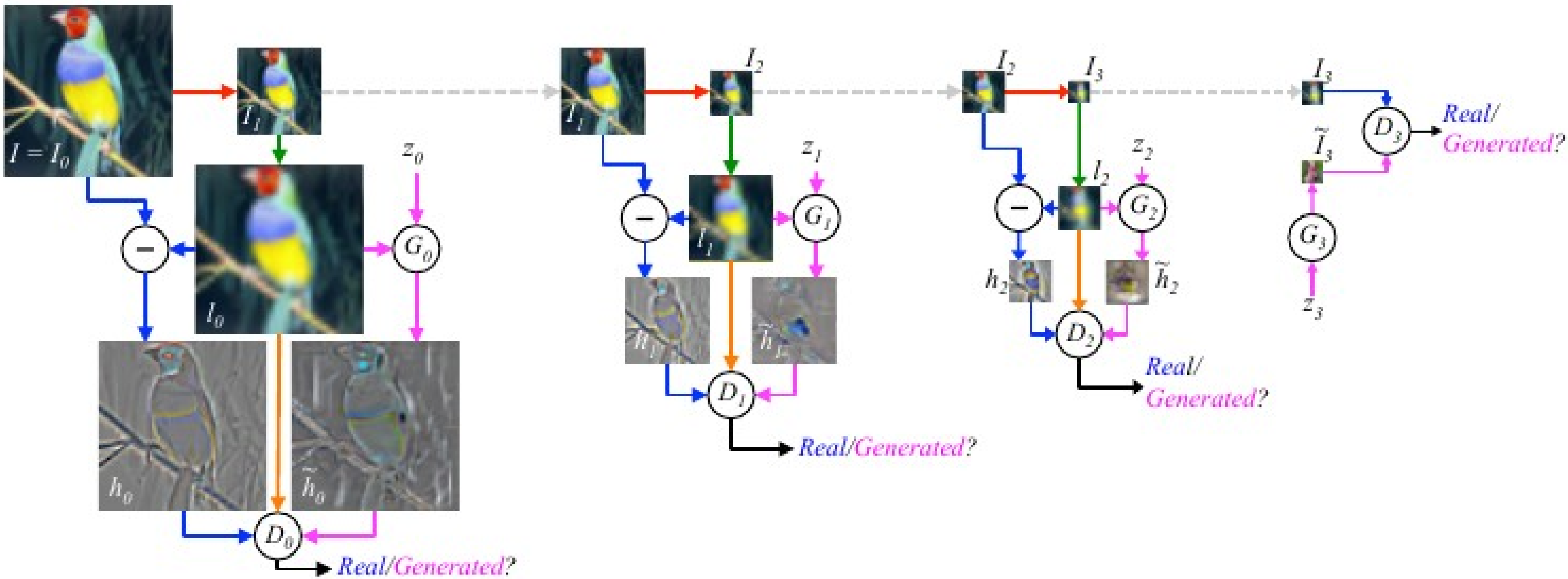
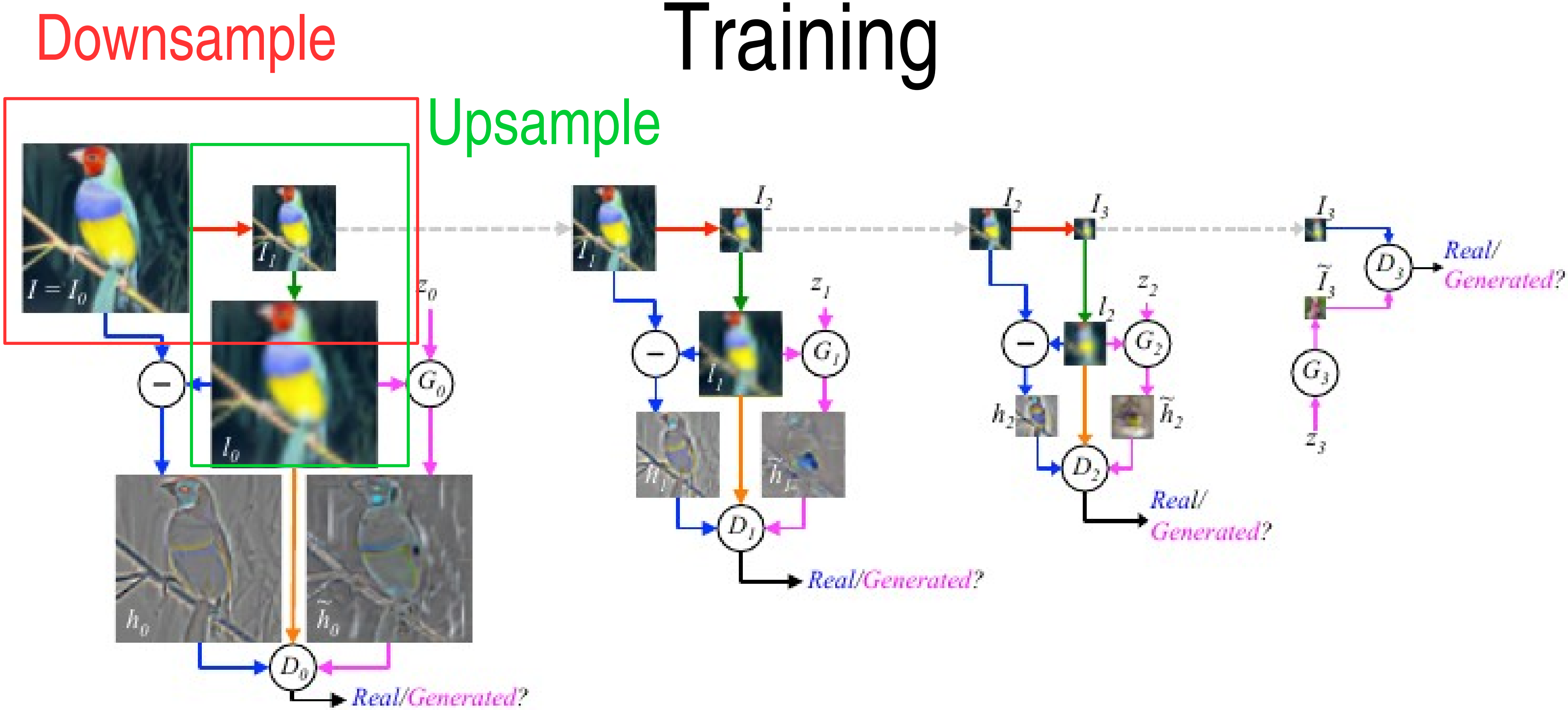


Figure 2: The training procedure for our LAPGAN model. Starting with a 64x64 input image I from our training set (top left): (i) we take $I_0 = I$ and blur and downsample it by a factor of two (red arrow) to produce I_1; (ii) we upsample I_1 by a factor of two (green arrow), giving a low-pass version l_0 of I_0; (iii) with equal probability we use l_0 to create *either* a real *or* a generated example for the discriminative model D_0. In the real case (blue arrows), we compute high-pass $h_0 = I_0 - l_0$ which is input to D_0 that computes the probability of it being real vs generated. In the generated case (magenta arrows), the generative network G_0 receives as input a random noise vector z_0 and l_0. It outputs a generated high-pass image $\tilde{h}_0 = G_0(z_0, l_0)$, which is input to D_0. In both the real/generated cases, D_0 also receives l_0 (orange arrow). Optimizing Eqn. 2, G_0 thus learns to generate realistic high-frequency structure $\tilde{h}_0$ consistent with the low-pass image l_0. The same procedure is repeated at scales 1 and 2, using I_1 and I_2. Note that the models at each level are trained independently. At level 3, I_3 is an 8×8 image, simple enough to be modeled directly with a standard GANs G_3 & D_3.

Training



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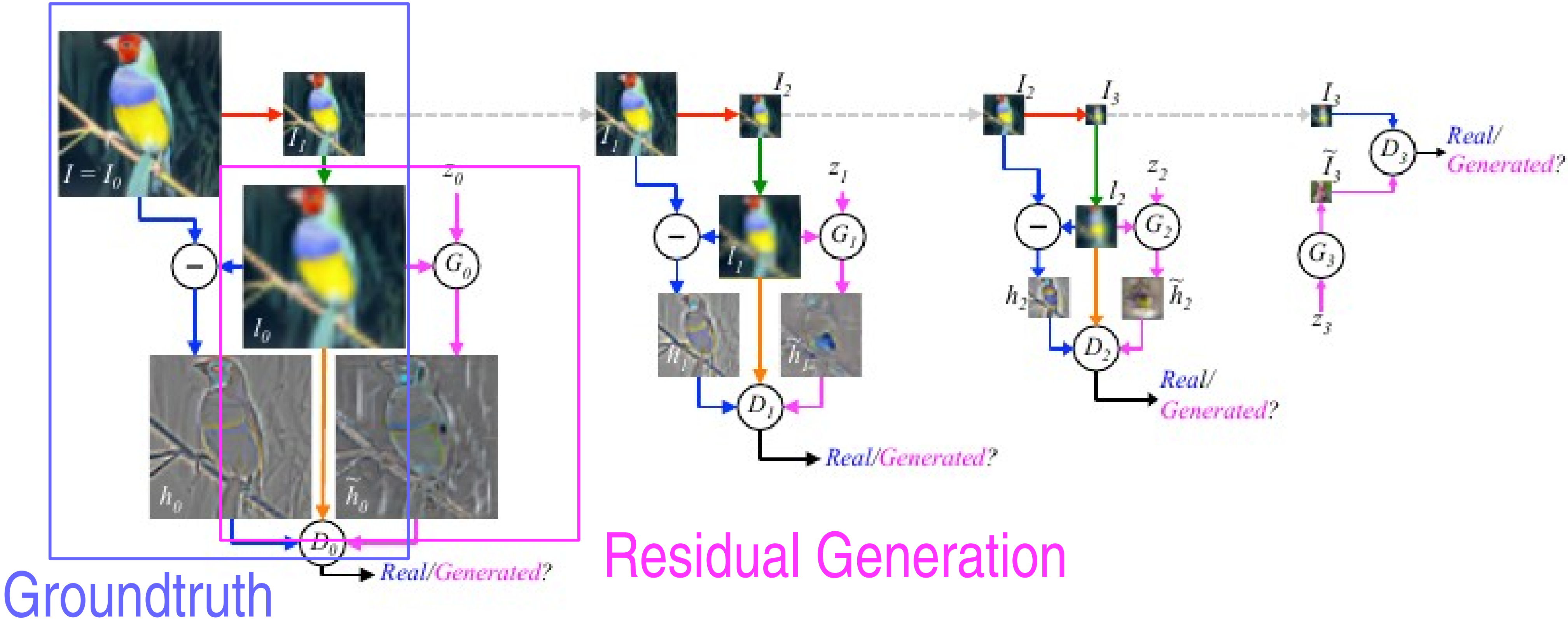

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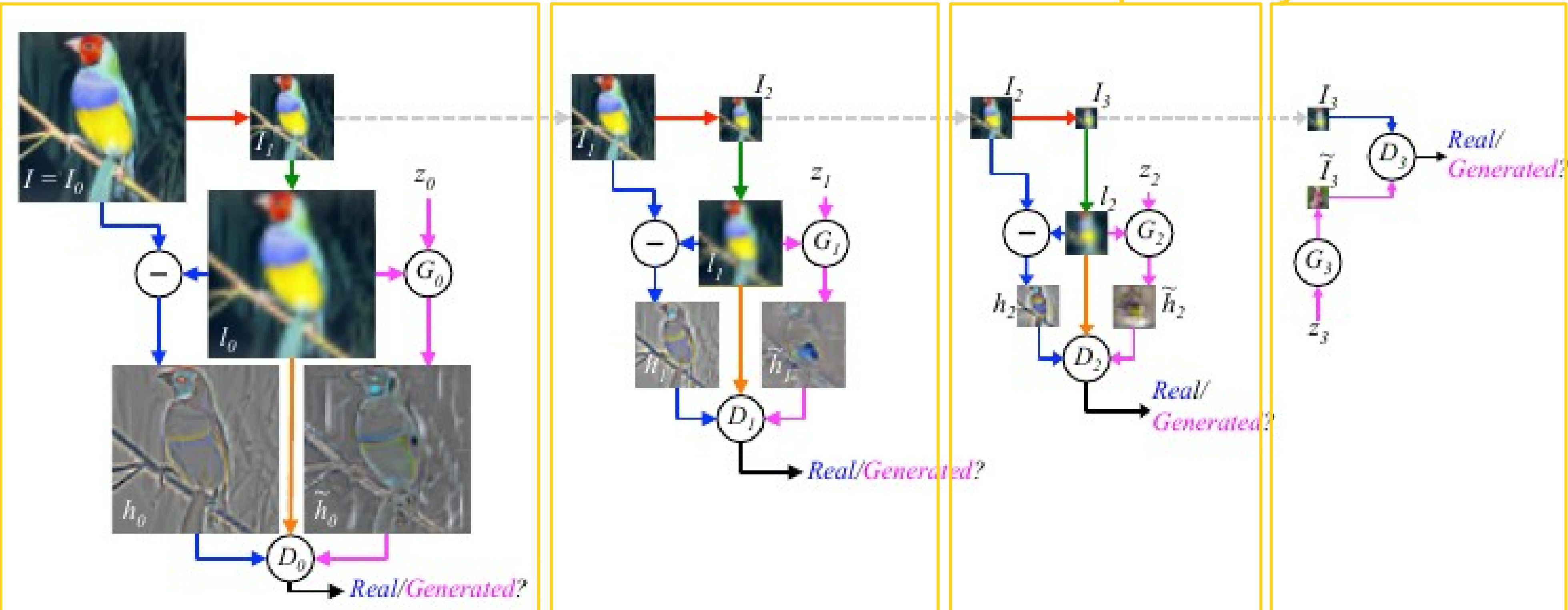
Training

Each step trained separately



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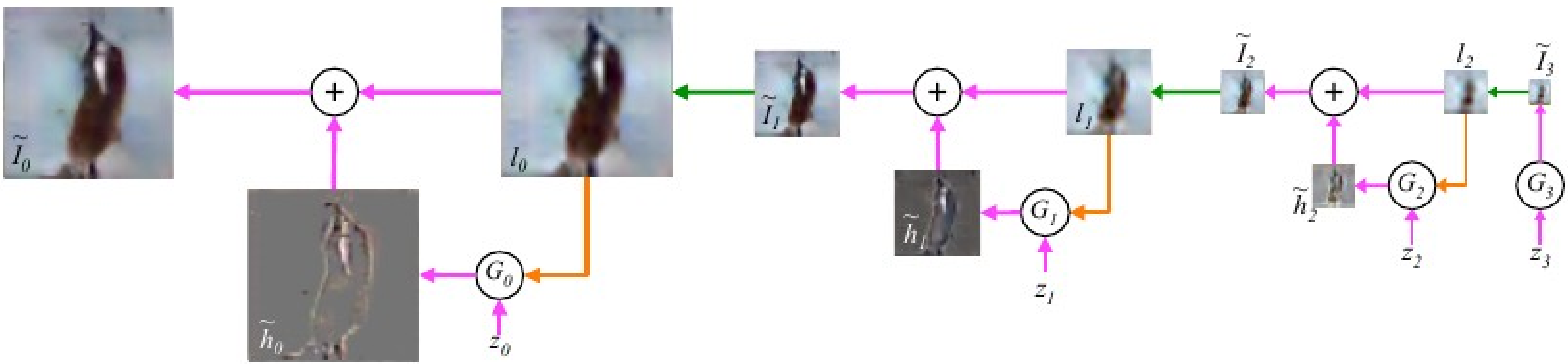
Generation



$\tilde{I}_0$
+
I_0
$\tilde{h}_0$
G_0
z_0
$\tilde{I}_1$
+
I_1
$\tilde{h}_1$
G_1
z_1
$\tilde{I}_2$
+
I_2
$\tilde{I}_3$
$\tilde{h}_2$
G_2
z_2
G_3
z_3

Results